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**Concrete Mathematics - MA 201**
**Problem Sheet - 3**

1. Find $J(20), J(40), J(80)$. In general, find $J(2^m \cdot 5)$, for any positive integer $m$.
2. For any positive integer $m$, find $J(2^m)$. (OR) Can we say that the first person will survive whenever $n$ is a power of 2?
3. Find the survivor's numbers of all even numbers upto 16.
4. Find the survivor's numbers of all odd numbers upto 15.
5. Using the recurrence relation
$$\begin{aligned} J(1) &= 1 \\ J(2n) &= 2J(n) - 1, \quad \text{for} \quad n \geq 1 \\ J(2n+1) &= 2J(n) + 1, \quad \text{for} \quad n \geq 1 \end{aligned}$$
find $J(42), J(39)$ and $J(61)$.
6. Prove that any positive integer $n$ can be written in the form
$$n = 2^m + \ell$$
where $2^m$ is the largest power of 2 not exceeding $n$ and where $\ell$ is remainder, $0 \leq \ell \leq 2^m - 1$.
7. Using induction, prove that
$$J(2^m + \ell) = 2\ell + 1$$
where $2^m$ is the largest power of 2 not exceeding $n$ and where $\ell$ is remainder, $0 \leq \ell \leq 2^m - 1$.. Also find $J(102)$. [Note that the induction is on $m$.]
8. Find $r$ such that $(121)_r = (144)_8$ where $r$ and 8 are the bases.
9. Find $J(343)$ by using binary notation. Analyse the case when $b_{m-1} = 0$.
10. Is the following statement correct?

    If we start with $n$ and iterate the $J$ function $m+1$ times $\big($applying $J$ repeatedly with itself, we get $J(n), J^2(n), J^3(n), \ldots, J^{m+1}(n)\big)$, then we end up with $n$ again. Note that each $n$ is an $(m+1)$-bit number and we are doing $m+1$ one-bit cyclic left-shifts.
11. Prove that for a given integer $n$, the following are equivalent:
    (a) $J(n) = n$ (thus, $n = J(n) = J^2(n) = \cdots$).
    (b) All bits of $n$ are 1.
    (c) $n = 2^{m+1} - 1$, where $n = 2^m + \ell, \quad 0 \leq \ell \leq 2^m - 1$.

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