Department of Mathematical and Computational Sciences, NITK, Surathkal
Linear Algebra and Matrices (MA 204)
https://sam.nitk.ac.in/sites/default/MA204-one.pdf

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Problem Sheet 1

1. Construct a 3×3 nonzero matrix A such that the vector $\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$ is a solution of $Ax = 0$.

2. Construct three different linear systems $Ax = b$ whose solution set is $x_1 = -2$, $x_2 = 1$ and $x_3 = 0$.

3. Find all the values of a and b if the following systems
$$x + y = 1; \qquad 2x + ay = b$$
   (a) has only one solution,
   (b) infintely many solutions and
   (c) no solution.

4. Choose h and k such that the system
$$x + hy = 2; \quad 4x + 8y = k$$
   (a) has only one solution,
   (b) infintely many solutions and
   (c) no solution.

5. Under what condition on y_1, y_2, y_3 do the points $(0, y_1), (1, y_2), (2, y_3)$ lie on a straight line. What is an appropriate generalization of the result?

6. Use Gaussian elimination to find a polynomial which passes through the following points (0,0), (1,4), (1,0) and (-2,10).

7. If (a, b) is a multiple of (c, d) with $abcd \neq 0$, show that (a, c) is a multiple of (b, d).

8. Find $PA = LU$ factorization of the matrix $\begin{pmatrix} -5 & 3 & 4 \\ 0 & 0 & -9 \\ 15 & 1 & 2 \end{pmatrix}$.

9. Factor the following *tridiagonal* matrices into LU and LDU : $\begin{pmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix}$ and $\begin{pmatrix} a & a & 0 \\ a & a+b & b \\ 0 & a & b+c \end{pmatrix}$.

10. If $A = \begin{pmatrix} 1 & -2 \\ -2 & 5 \end{pmatrix}$ and $AB = \begin{pmatrix} -1 & 2 & -1 \\ 6 & -9 & 3 \end{pmatrix}$, determine the first and second columns of B.