Department of Mathematical and Computational Sciences, NITK, Surathkal
Linear Algebra and Matrices (MA 204)
https://sam.nitk.ac.in/sites/default/MA204-ps-3.pdf

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Problem Sheet 3

1. Say TRUE or FALSE with justification.
   (a) Let A be an $n \times n$ matrix and $Ax = b$ has a solution. Then $A^T x = b$ also has a solution.
   (b) If $B = A^{-1}$ then solvability of $Ax = b$ implies solvability of $Bx = b$.
   (c) If each $Ax = b$ and $Bx = b$ has a solution, then $(A + B)x = b$ has a solution.
   (d) Let $Ax = b$ be solvable. If $r(A) \leq r(B)$ then $Bx = b$ is also solvable. Here $r(A)$ denotes the rank of the matrix A.
   (e) The product of permutation matrices is again a permutation matrix.
   (f) The inverse of a permutation matrix is itself.
   (g) There are 2^n permutations of size n.
   (h) For any square matrix A, $C(A) \subseteq C(A^2)$ and $N(A) \subseteq N(A^2)$. The column and null spaces of A are denoted by $C(A)$ and $N(A)$ respectively.
2. (a) What is the column space of an invertible $n \times n$ matrix? What is the nullspace of that matrix?
   (b) If every column of A is a multiple of the first column, what is the column space of A?
3. Multiplying a matrix A times the column vector $x = (2, 1)$ gives what combination of the columns of A? How many rows and columns in A?
4. If A is the 2×2 matrix $\begin{pmatrix} 2 & 4 \\ 7 & 3 \end{pmatrix}$, what are its pivots?
5. Let $A = \begin{pmatrix} 2 & 1 \\ 6 & 3 \end{pmatrix}$. Find b and c so that $Ax = b$ has no solution and $Ax = c$ has a solution.
6. How can you find the inverse of A by working with $[A \quad I]$? If you solve the n equations $Ax =$ columns of I then the solutions x are columns of __________.
7. Give an example of vectors that span $\mathbb{R}^2$ but are not a basis for $\mathbb{R}^2$. Also write down a basis for $\mathbb{R}$.
8. Is there a 3×3 permutation matrix P, besides $P = I$, such that $P^3 = I$? If yes, give one such P. If no, explain why?
9. Find the right inverse of $\begin{pmatrix} 4 & 0 & 0 \\ 0 & 5 & 0 \end{pmatrix}$.
10. Find $PA = LDU$ factorization of $\begin{pmatrix} 3 & -5 & 1 \\ 1 & 1 & 1 \\ -1 & 5 & -2 \\ 3 & -7 & 8 \end{pmatrix}$.