

Instructor : P. Sam Johnson

Problem Sheet : Basic Probability

1. Find the probability that in tossing a fair coin three times. there will appear (a) 3 heads, (b) 2 tails and 1 head, (c) at least 1 head, (d) not more than 1 tail.
2. Find the probability that in five tosses of a fair die, a 3 will appear (a) twice, (b) at most once, (c) at least two times.
3. Find the probability that in a family of 4 children there will be (a) at least 1 boy, (b) at least 1 boy and at least 1 girl. Assume that the probability of a male birth is $1/2$.
4. Out of 2000 families with 4 children each, how many would you expect to have (a) at least 1 boy, (b) 2 boys, (c) 1 or 2 girls, (d) no girls?
5. If 20% of the bolts produced by a machine are defective, determine the probability that out of 4 bolts chosen at random, (a) 1, (b) 0, (c) less than 2, bolts will be defective.
6. Find the probability of getting a total of 7 at least once in three tosses of a pair of fair dice.
7. Prove that the mean and variance of a binomially distributed random variable are, respectively, $\mu = np$ and $\sigma^2 = npq$.
8. If the probability of a defective bolt is 0.1, find (a) the mean, (b) the standard deviation, for the number of defective bolts in a total of 400 bolts.
9. Show that the mean and variance of the uniform distribution are given respectively by (a) $\mu = \frac{1}{2}(a + b)$ (b) $\sigma^2 = \frac{1}{12}(b - a)^2$.
10. The Probability that an entering college student will graduate is 0.4. Determine the probability that out of 5 students (a) none, (b) at least 1, will graduate.
11. What is the probability of getting a total of 9 (a) twice, (b) at least twice in 6 tosses of a pair of dice?
12. If the probability of a defective bolt is 0.1, find (a) the mean, (b) the standard deviation, for the distribution of defective bolts in a total of 400.
13. Prove that $\binom{n}{r} = \binom{n-1}{r} + \binom{n-1}{r-1}$.
14. Find the constant term in the expansion of $\left(x^2 + \frac{1}{x}\right)^{12}$.