Department of Mathematical and Computational Sciences, NITK, Surathkal
Probability Thoery and Applications (MA 208)
https://sam.nitk.ac.in/sites/default/MA208-ps-2.pdf

Instructor : P. Sam Johnson

Problem Sheet : Characterizations of Randorm Vairables

1. Suppose that a game is to be played with a single die assumed fair. In this game a player wins \$20 if a 2 turns up, \$40 if a 4 turns up; loses \$30 if a 6 turns up; while the player meither wins nor loses if any other face turns up. Find the expected sum of money to be won.

2. In a lottery there are 200 prizes of \$5, 20 prizes of \$25, and 5 prizes of \$100. Assuming that 10,000 tickets are to be issued and sold, what is a fair price to pay for tickets.

3. Find the expectation of the sum of points in tossing a pair of fair dice.

4. Find the expectation of a discrete random variable X whose probability function is given by
$$f(x) = \left(\frac{1}{2}\right)^x \quad (x = 1, 2, 3, \ldots)$$

5. A continuous random variable X has porbability density given by
$$f(x) = \begin{cases} 2e^{-2x} & x > 0 \\ 0 & x \leq 0 \end{cases}$$
Find (a) $E(X)$, (b) $E(X^2)$.

6. The joint density function of two random variables X and Y is given by
$$f(x, y) = \begin{cases} xy/96 & 0 < x < 4, 1 < y < 5 \\ 0 & \text{otherwise} \end{cases}$$
Find (a) $E(X)$, (b) $E(Y)$, (c) $E(XY)$, (d) $E(2X + 3Y)$.

7. Find (a) the variance, (b) the standard deviation of the sum obtained in tossing a pair of fair dice.

8. Find (a) the variance, (b) the standard deviation for the random variable of Problem(5)

9. Prove the quantity $E[(X - a)^2]$ is minimum when $a = \mu = E(X)$.

10. If $X^* = (X - \mu)/\sigma$ is standardized random variable, prove that (a) $E(X^*) = 0$, (b) $\text{Var}(X^*) = 1$

11. The joint probability function of two discrete random variables X and Y si given by $f(x, y) = c(2x + y)$, where x and y can assume all integers such that $0 \leq x \leq 2, 0 \leq y \leq 3$, and $f(x, y) = 0$ otherwise. Find the conditional expectation of Y given $X = 2$.

12. The joint density function of the random variables X and Y is given by
$$f(x, y) = \begin{cases} 8xy & 0 \leq x \leq 1, 0 \leq y \leq x \\ 0 & \text{otherwise} \end{cases}$$
Find the conditional expectation of (a) Y given X, (b) X given Y.

13. Find the conditional variance of Y given X for problem (12).

14. A random vairable X has density function given by
$$f(x) = \begin{cases} 2e^{-2x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$
(a) find $P(|X - \mu| > 1)$. (b) Use Chebyshev's inequality to obtain an upper bound on $P(|X - \mu| > 1)$ and compare with the result in (a).

15. Prove that $-1 \leq \rho \leq 1$, where ρ is the correlation coefficient.