

Instructor : P. Sam Johnson

Problem Sheet
Gamma Distribution

1. Show that the mean and variance of the gamma distribution are given by
 - (a) $\mu = r/\alpha$,
 - (b) $\sigma^2 = r/\alpha^2$.
2. Show that $\Gamma(\frac{1}{2}) = \sqrt{\pi}$.

Chi-square Distribution

1. Let X be a normally distributed random variable having mean 0 and variance 1. Show that X^2 is chi square distributed with 1 degrees of freedom.
2. Let X_1 and X_2 be independent random variables that are chi square distributed with ν_1 and ν_2 degrees of freedom, respectively. Show that the moment generating function of $Z = X_1 + X_2$ is $(1 - 2t)^{-(\nu_1 + \nu_2)/2}$, thereby, show that Z is chi square distributed with $\nu_1 + \nu_2$ degrees of freedom.
3. Let X_1, X_2 be independent normally distributed random variables with mean 0 and variance 1. Then $\chi^2 = X_1^2 + X_2^2$ is chi square distributed with 2 degrees of freedom. [Hint: Use problems 1 and 2]
4. The graph of the chi-square distributed with 5 degrees of freedom is shown below. Find the values χ_1^2, χ_2^2 for which
 - (a) the shaded area on the right = 0.05,
 - (b) the total shaded area = 0.05,
 - (c) the shaded area on the left = 0.10,
 - (d) the shaded area on the right 0.01.
5. Find the values of χ^2 for which the area of the right-hand tail of the χ^2 distribution is 0.05, if the number of degrees of freedom ν is equal to
 - (a) 15,
 - (b) 21,
 - (c) 50.
6. Suppose that the random variable X has a chi-square distribution with 10 degrees of freedom. If we are asked to find two numbers a and b such that $P(a < x < b) = 0.85$, say, we should realize that there are many pairs of this kind.
 - (a) Find two different sets of values (a, b) satisfying the above condition.
 - (b) Suppose that in addition to the above, we require that

$$P(X < a) = P(X > b).$$

How many sets of values are there?