Department of Mathematical and Computational Sciences, NITK, Surathkal
Probability Theory and Applications (MA 208)

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Problem Sheet
Moment Generating Function

1. The random variable X can assume the values 1 and -1 with probability $\frac{1}{2}$ each. Find
   (a) the moment generating function
   (b) the first four moments about the origin.

2. A random vairable X has density function given by
$$f(x) = \begin{cases} 2e^{-2x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$
   Find the moment generating function and the first four moments about the origin.

3. Find the first four moments about the origin and about the mean, for a random variable X having density function
$$f(x) = \begin{cases} 4x(9-x^2)/81 & 0 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

4. If $M(t)$ is the moment generating function for a random variable X, prove that the mean is $\mu = M'(0)$ and the variance is $\sigma^2 = M''(0) - [M'(0)]^2$.

5. Find the moment generating function of a random vairable X that is binomially distributed.

6. Find the moment generating function for the general normal distribution.

7. Show that the moment generating function of a random variable X, which is chi square distributed with v degrees of freedom, is $M(t) = (1-2t)^{-\nu/2}$.

8. Let X_1 and X_2 be independent random variables that are chi square distributed with ν_1 and ν_2 degrees of freedom, respectively.
   (a) Show that the moment generating function of $Z = X_1 + X_2$ is $(1-2t)^{-(\nu_1+\nu_2)/2}$, thereby
   (b) show that Z is chi square distributed with $\nu_1 + \nu_2$ degrees of freedom.

9. Suppose that X has pdf given by
$$f(x) = 2x, \quad 0 \leq x \leq 1.$$
   (a) Determine the mgf of X.
   (b) Using the mgf, evaluate $E(X)$ and $V(X)$ and check your answer.

10. Suppose that S, a random voltage, varies between 0 and 1 volt and is uniformly distributed over that interval. Suppose that the signal S is perturbed by an additive, independent random noise N which is uniformly distributed between 0 and 2 volts.
    (a) Find the mgf of the voltage (including noice).
    (b) Using the mgf, obtain the expected value and variance of this voltage.

11. Let X be the outcome when a fair die is tossed.
    (a) Find the mgf of X.
    (b) Using the mgf, find $E(X)$ and $V(X)$.

12. Suppose that X has the following pdf:

$$f(x) = \lambda e^{-\lambda(x-a)}, \quad x \geq a.$$

This is known as a **two-parameter exponential distribution.**

(a) Find the mgf of X.

(b) Using the mgf, find $E(X)$ and $V(X)$.

13. Suppose that the continuous random variables X had pdf

$$f(x) = e^{-|x|}/2, \quad -\infty < x < \infty.$$

(a) Obtain the mgf of X.

(b) Using the mgf, find $E(X)$ and $V(X)$.

14. Suppose that the mgf of a random variable X is of the form

$$M_X(t) = (0.4e^t + 0.6)^8.$$

(a) What is the mgf of the random variable $Y = 3X + 2$?

(b) Evaluate $E(X)$.

(c) Can you check your answer to (14b) by some other method?

15. In a circuit n resistances are hooked up into a series arrangement. Suppose that each resistance is uniformly distributed over $[0, 1]$ and suppose, furthermore, that all resistances are independent. Let R be the total resistance.

(a) Find the mgf of R.

(b) Using the mgf, obtain $E(R)$ and $V(R)$. Check your answers by direct computation.

16. If X has distribution χ^2_n, using the mgf, show that $E(X) = n$ and $V(X) = 2n$.

17. Suppose that V, the velocity (cm/sec) of an object, has distribution $N(0, 4)$. If $K = mV^2/2$ ergs is the kinetic energy of the object (where m = mass), find the pdf of K. If $m = 10$ grams, evaluate $P(K \leq 3)$.

18. Suppose that $X_1, \ldots, X_{80}$ are independent random variables, each having distribution $N(0, 1)$. Evaluate $P[X_1^2 + \cdots + X_{80}^2 > 77]$.

19. A number of resistances, R_i, $i = 1, 2, \ldots, n$, are put into a series arrangement in a circuit. Suppose that each resistance is normally distributed with $E(R_i) = 10$ ohms and $V(R_i) = 0.16$.

(a) If $n = 5$, what is the probability that the resistance of the circuit exceeds 49 ohms?

(b) How large should n be so that the probability that the total resistance exceeds 100 ohms is approximately 0.05?

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