Department of Mathematical and Computational Sciences, NITK, Surathkal
Probability Theory and Applications (MA 208)

Instructor : P. Sam Johnson

Problem Sheet
Normal Distribution

1. Use the mgf to show that if X and Y are independent random variables with distribution $N(\mu_x, \sigma_x^2)$ and $N(\mu_y, \sigma_y^2)$, respectively, then $Z = aX + bY$ is again normally distributed, where a and b are constants.

2. Find the area under the standard normal curve

   (a) between $z = 0$ and $z = 1.2$,

   (b) between $z = -0.68$ and $z = 0$,

   (c) between $z = -0.46$ and $z = 2.21$,

   (d) between $z = 0.81$ and $z = 1.94$,

   (e) to the right $z = -1.28$.

3. If "area" refers to that under the standard normal curve, find the value or values of z such that

   (a) area between 0 and z is 0.3770,

   (b) area to left of z is 0.8621,

   (c) area between -1.5 and z is 0.0217

4. The mean weight of 500 male students at a certain college is 151 lb and the standard deviation is 15 lb. Assuming that the weights are normally distributed, find how many students weigh

   (a) between 120 and 155 lb,

   (b) more than 185 lb.

5. The mean inside diameter of a sample of 200 washers produced by a machine is 0.502 inches and the standard deviation is 0.005 inches. the purpose for which these washers are intended allows a maximum tolerance in the diameter of 0.496 to 0.508 inches, otherwise the washers are considered defective. Determine the percentage of defective washers prodeced by the machine, assuming the diameters are normally distributed.

6. Find the probability of getting between 3 **and** 6 **heads inclusive in** 10 **tosses** of a fair coin by using

   (a) the binomial distribution,

   (b) the normal approximation to the binomial distribution.

7. A fair coin is tossed 500 times. Find the probability that the number of heads will not differ from 250 by

   (a) more than 10,

   (b) more than 30.

8. A die is tossed 120 times. Find the probability that the face 4 will turn up

   (a) 18 times or less

   (b) 14 times or less, assuming that the die is fair.

9. Suppose that X has distribution $N(2, 0.16)$. Using the table of the normal distribution, evaluate the following probabilities.

(a) $P(X \geq 2.3)$

(b) $P(1.8 \leq X \leq 2.1)$.

10. The diameter of an electric cable is normally distributed with mean 0.8 and variance 0.0004.

    (a) What is the probability that the diameter will exceed 0.81 inch?

    (b) Suppose that the cable is considered defective if the diameter differs from its mean by more than 0.025. What is the probability of obtaining a defective cable?

11. The errors in a certain length-measuring device are known to be normally distributed with expected value zero and standard deviation 1 inch. What is the probability that the error is measurement will be greater than 1 inch? 2 inches? 3 inches?

12. Suppose that the life lengths of two electronic devices, say D_1 and D_2, have distribution $N(40, 36)$ and $N(45, 9)$ respectively. If the electronic device is to be used for a 45-hour period, which device is to be preferred? If it is to be used for a 48-hour period, which device is to be preferred?

13. Let $Y = |X|$. If X has distribution $N(0, 1)$, determine the pdf of Y, and evaluate $E(Y)$ and $V(Y)$.

14. Suppose that we are measuring the position of an object in the plane. Let X and Y be the errors of measurement of the x- and y-coordinates, respectively. Assume that X and Y are independently and identically distributed, each with distribution $N(0, \sigma^2)$.

    (a) Find the pdf of $R = \sqrt{X^2 + Y^2}$. The distribution of R is known as the **Rayleigh distribution.** [Hint: Let $X = R\cos\phi$ and $Y = R\sin\phi$. Obtain the joint pdf of (R, ϕ) and then obtain the marginal pdf of R.]

    (b) Find the pdf of the random variable $Q = X/Y$, where X and Y are distributed. The distribution of Q is known as the **Cauchy distribution.** Can you compute $E(Q)$?

15. A distribution closely related to the normal distribution is the **lognormal** distribution. Suppose that X is normally distributed with mean μ and variance σ^2. Let $Y = e^X$. Then Y has the lognormal distribution. (That is, Y is lognormal if and only if Y is normal.) Find the pdf of Y.

16. Suppose that temperature (measured in degrees centrigrade) is normally distributed with expectation 50° and variance 4. What is the probability that the temperature T will be between 48° and 53° centrigrade?

17. Suppose that X has distribution $N(\mu, \sigma^2)$. Determine c (as a function of μ and σ) such that $P(X \leq c) = 2P(X > c)$.

18. The outside diameter of a shaft, say D, is specified to be 4 inches. Consider D to be a normally distributed random variable with mean 4 inches and variance 0.01 inch^2. If the actual diameter differs from the specified value by more than 0.05 inch but less than 0.08 inch, the loss to the manufacturer is \$0.50. If the actual diameter differs from the specified diameter by more than 0.08 inch, the loss is \$1.00. The loss, L, may be considered as a random variable. Find the probability distribution of L and evaluate $E(L)$.

19. The grades on a short quiz in biology were $0, 1, 2, \ldots, 10$ points, depending on the number answered correctly out of 10 questions. The mean grade was 6.7, and the standard deviation was 1.2. Assuming the grades to be normally distributed, determine

    (a) the percentage of students scoring 6 points,

    (b) the maximum grade of the lowest 10% of the class,

    (c) the minimum grade of the highest 10% of the class.

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Standard Normal Cumulative Probability Table

Cumulative probabilities for NEGATIVE z-values are shown in the following table:

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Standard Normal Cumulative Probability Table

Cumulative probabilities for POSITIVE z-values are shown in the following table:

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998