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**Problem Sheet**  
**Some Important Discrete Random Variables**

1. Establish the validity of the Poisson approximation to the binomial distribution.
2. Ten percent of the tools produced in a certain manufacturing process turn out to be defective. Find the probability that in a sample of 10 tools chosen at random, exactly 2 will be defective, by using
  - (a) the binomial distribution,
  - (b) the Poisson approximation to the binomial distribution.
3. If  $X$  has a Poisson distribution with parameter  $\beta$ , and if  $P(X = 0) = 0.2$ , evaluate  $P(X > 2)$ .
4. Let  $X$  have a Poisson distribution with parameter  $\lambda$ . Find that value of  $k$  for which  $P(X = k)$  is largest. [Hint; Compare  $P(X = k)$  with  $P(X = k - 1)$ .]
5. Suppose that  $X$  has a Poisson distribution. If  $P(X = 2) = \frac{2}{3}P(X = 1)$ . evaluate  $P(X = 0)$  and  $P(X = 3)$ .
6. Suppose that a container contains, 10,000 particles. The probability that such a particle escapes from the container equals 0.0004. What is the probability that more than 5 such escapes occur? (You may assume that the various escapes are independent of one another.)
7. Suppose that a book of 585 pages contains 43 typographical errors. If these errors are randomly distributed throughout the book, what is the probability that 10 pages, selected at random, will be free of errors? [Hint: Suppose that  $X =$  number of errors per page has a Poisson distribution.]
8. Define Pascal distribution. Find its mean and variance.
9. A machinist keeps a large number of washers in a drawer. About 50 percent of these washers are  $\frac{1}{4}$  inch in diameter, about 30 percent are  $\frac{1}{8}$  inch in diameter, and the remaining 20 percent are  $\frac{3}{8}$  inch in diameter. Suppose that 10 washers are chosen at random.
  - (a) What is the probability that there are exactly five  $\frac{1}{4}$ -inch washers, four  $\frac{1}{8}$ -inch washers, and one  $\frac{3}{8}$ -inch washer?
  - (b) What is the probability that only two kinds of washers are among the chosen ones?
  - (c) What is the probability that all three kinds of washers are among the chosen ones?
  - (d) What is the probability that there are three of one kind, three of another kind, and four of the third kind in a sample of 10?
10. If the probability that an individual will suffer a bad reaction from injection of a given serum (the component that is neither blood cell nor a clotting factor) is 0.001, determine the probability that out of 2000 individuals,
  - (a) exactly 3,
  - (b) more than 2, individuals will suffer a bad reaction.
11. Find the probability that in successive tosses of a fair die, a 3 will come up for the first time on the fifth toss.
12. A box contains 6 blue marbles and 4 red marbles. An experiment is performed in which a marble is chosen at random and its color observed, but the marble is not replaced. Find the probability that after 5 trials of the experiment, 3 blue marbles will have been chosen.
13. A box contains 5 red balls, 4 white balls, and 3 blue balls. A ball is selected at random from the box, its color is noted, and then the ball is replaced. Find the probability that out of 6 balls selected in this manner, 3 are red, 2 are white, and 1 is blue.

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