Department of Mathematical and Computational Sciences, NITK, Surathkal
Probability Theory and Applications (MA 208)

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Problem Sheet
Some Important Continuous Randorm Vairables
Exponential, Gamma and Chi-square Distributions

1. Show that the mean and variance of the exponential distribution are given by

   (a) $E(X) = 1/\alpha$, (b) $V(X) = 1/\alpha^2$.

2. The exponential and geometric distributions have the property of having "no memory (lack-of-memory)". What does it mean?

3. Say true or false. The only continuous random variable assuming nonnegative values having "no memory (lack-of-memory)" property is an exponentially distributed random variable.

4. Show that the mean and variance of the gamma distribution are given by

   (a) $\mu = r/\alpha$, (b) $\sigma^2 = r/\alpha^2$.

5. Show that $\Gamma(\frac{1}{2}) = \sqrt{\pi}$.

6. Let X be a normally distributed random variable having mean 0 and variance 1. Show that X^2 is chi square distributed with 1 degrees of freedom.

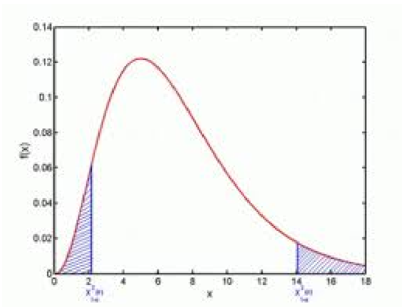
7. Let X_1 and X_2 be independent random variables that are chi square distributed with ν_1 and ν_2 degrees of freedom, respectively. Show that the moment generating function of $Z = X_1 + X_2$ is $(1-2t)^{-(\nu_1+\nu_2)/2}$, thereby, show that Z is chi square distributed with $\nu_1 + \nu_2$ degrees of freedom.

8. Let X_1, X_2 be independent normally distributed random variables with mean 0 and variance 1. Then $\chi^2 = X_1^2 + X_2^2$ is chi squre distributed with 2 degrees of freedom. [Hint: Use problems 6 and 7]

9. The graph of the chi-square distributed with 5 degrees of freedom is shown below. Find the values χ_1^2, χ_2^2 for which

   (a) the shaded area on the right $= 0.05$, (c) the shaded area on the left $= 0.10$,

   (b) the total shaded area $= 0.05$, (d) the shaded area on the right 0.01.



10. Find the values of χ^2 for which the area of the right-hand tail of the χ^2 distribution is 0.05, if the number of degrees of freedom ν is equal to

    (a) 15, (b) 21, (c) 50.

11. Suppose that the random variable X has a chi-square distribution with 10 degrees of freedom. If we are asked to find two numbers a and b such that $P(a < x < b) = 0.85$, say, we should realize that there are many pairs of this kind.

    (a) Find two different sets of values (a, b) satisfying the above condition.

    (b) Suppose that in addition to the above, we require that

    $$P(X < a) = P(X > b).$$

    How many sets of values are there?

12. Compare the **upper bound** on the probability $P[|X - E(X)|] \geq 2\sqrt{V(X)}$ obtained from Chebyshev's inequality with the exact probability in each of the following cases.

    (a) X has distribution $N(\mu, \sigma^2)$.

    (b) X has Poisson distribution with parameter λ.

    (c) X had exponential distribution with parameter α.

13. Suppose that X is a random variable for which $E(X) = \mu$ and $V(X) = \sigma^2$. Suppose that Y is uniformly distributed over the interval (a, b). Determine a and b so that $E(X) = E(Y)$ and $V(X) = V(Y)$.

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