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Computational Linear Algebra - MA 703
Problem Sheet 7

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1. Say true or false with justification: Let A be a matrix with real entries. Then the set of all eigenvalues of A is always nonempty.

2. Find the matrix associated with the quadratic form $p(x, y, z) = 2x^2 - 7y^2 + 3z^2 - 2xy - yz$.

3. If A is a real symmetric matrix, then prove that the eigenvectors corresponding to distinct eigenvalues are pairwise orthogonal.

4. What are the eigenvalues of the rank one matrix $[1\ 2\ 1]^T[1\ 1\ 1]$?

5. If $A = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$, then find the followings :

   (a) the eigenvalues and the eigenvectors of A

   (b) find the matrix P such that $P^{-1}AP$ is a diagonal matrix (diagonalization of A), and

   (c) compute A^n for each positive integer n.

6. If the eigen values of a 3×3 matrix B are $0, 1, 2$, then find the followings :

   (a) the rank of B

   (b) the dimension of nullspace of B

   (c) the determinant of B^TB

   (d) the trace (the sum of diagonals) of $(B + B^T)$, and

   (e) the eigenvalues of $(B + I)^{-1}$.

7. Let A, B, C be $n \times n$ matrices. Assume that C is invertible and that $A^T = C^{-1}BC$. Which conclusion follows?

   (a) A and B have the same eigenvalues.

   (b) B and C have the same eigenvalues.

   (c) The eigenvalues of A are the negatives of the eigenvalues of B.

   (d) A^T and C have the same eigenvalues.

8. If 0 is an eigenvalue of the $n \times n$ matrix A, which conclusion is not justified?

   (a) A is the 0-matrix.

   (b) A is not invertible.

   (c) A has a nonzero null space (or kernel).

   (d) The rankk of A is less than n.

9. Let A be a matrix with real entries. Which of the following is true?

   (a) The set of all eigenvalues of A (the spectrum of A) is always nonempty.

   (b) If A^{-1} exists, the eigenvalues of A are real.

   (c) A is always diagonalizable.

   (d) If A is symmetric also, then the eigenvectors corresponding to eigenvalues are pairewise orthogonal.

10. Let $P = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}$, $D = \begin{bmatrix} 5 & 0 \\ 0 & 3 \end{bmatrix}$. Which gives A^k?

(a) $\begin{bmatrix} 2.5^k - 3^k & 5^k - 3^k \\ 2.3^k - 2.5^k & 2.3^k - 5^k \end{bmatrix}$

(b) $\begin{bmatrix} 2.5^k - 3^k & 2.5^k - c.3^k \\ 3^k - 5^k & -2.3^k - 5^k \end{bmatrix}$

(c) $\begin{bmatrix} 5^k & 0 \\ 0 & 3^k \end{bmatrix}$

(d) $\begin{bmatrix} 5^k & 5^k \\ -3^k & -2.3^k \end{bmatrix}$

11. Let A be a matrix (with real entries). Suppose that A has a complex eigenvalue λ. What conclusion can be drawn?

(a) A is upper triangular.
(b) A is not diagonalizable.
(c) A is not invertible.
(d) All eigenvectors of A corresponding to λ are complex.

12. If the columns of P are eigenvectors of A, what conclusion can be drawn?

(a) PA is a diagonal matrix.
(b) AP equals P times a digonal matrix.
(c) P is invertible and PAP^{-1} is diagonal.
(d) P is invertible and $P^{-1}AP$ is diagonal.

13. Let A and B be square matrices such that $AB = I$. Given only this information about A and B, which of these is an unjustified conclusion?

(a) 0 is not an eigenvalue of B.
(b) $BA = AB$.
(c) $\text{Det}(A) = 1/\text{Det}(B)$.
(d) The equation $(A - B)x = 0$ has only the zero solution, $x = 0$.

14. If $A = PDP^{-1}$, where D is diagonal and P is invertible, then

(a) A and P have the same eigenvalues.
(b) A and D have the same characteristic polynomial.
(c) A is invertible.
(d) D is invertible.

15. If a 3×3 matrix A has the characteristic equation $(x - 1)(x^2 - 4x + 3) = 0$, which conculsion is valid?

(a) A is diagonalizable.
(b) A is noninvertible.
(c) A is invertible.
(d) A is symmetric.

16. What are the eigenvalues of the rank one matrix $[1\ 2\ 1]^T[1\ 1\ 1]$?